

Measures of Spread - Standard Deviation

Recall the measures of spread (range, variance, standard deviation, and IQR).

Range is the difference between the maximum value and the minimum value of the data.

$$\text{Range} = \text{maximum value} - \text{minimum value}$$

A deviation is the difference between an individual value in a set of data and the mean for the data.

Variance is a measure of dispersion that is found by averaging the squares of the deviation of each piece of data.

$$\text{Var} = \frac{(x_1 - \bar{x})^2 + \dots + (x_n - \bar{x})^2}{n}$$

$$\text{Variance} = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}, \text{ where}$$

n - number of data values
 x_i - each value
 \bar{x} - mean

Standard Deviation is the measure of dispersion by taking the square root of the variance and is denoted by the Greek letter sigma, σ .

$$\sigma = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}} \quad \text{OR} \quad \sigma = \sqrt{\frac{(x_1 - \bar{x})^2 + \dots + (x_n - \bar{x})^2}{n}}$$

Standard Deviation for grouped data is calculated using

$$\sigma = \sqrt{\frac{\sum_{i=1}^n f_i (x_i - \bar{x})^2}{n}} \quad \text{OR} \quad \sigma = \sqrt{\frac{h(x_1 - \bar{x})^2 + \dots + f_n(x_n - \bar{x})^2}{n}}$$

Example 1: Use means and standard deviations to compare the distribution of heights for the following two basketball teams:

Falcons: 183, 165, 148, 146, 181, 178, 154

Panthers: 166, 163, 168, 161, 165, 166, 166

$$\bar{x} = \frac{183 + 165 + 148 + 146 + 181 + 178 + 154}{7} = 165$$

Solution:

Height (cm), h	$(x - \bar{x})$	$(x - \bar{x})^2$	Height (cm), h	$(h - \bar{x})$	$(h - \bar{x})^2$
183	$(183 - 165) = 18$	$18^2 = 324$			
165	$(165 - 165) = 0$	$0^2 = 0$			
148	$(148 - 165) = -17$	$(-17)^2 = 289$			
146	$(146 - 165) = -19$	$(-19)^2 = 361$			
181	$(181 - 165) = 16$	$(16)^2 = 256$			
178	$(178 - 165) = 13$	$(13)^2 = 169$			
154	$(154 - 165) = -11$	$(-11)^2 = 121$			

$$+ 1520$$

Example 2: Do the previous example using Excel or Fathom.

$$\text{Variance} = \frac{1520}{7}$$

$$\text{S.D.} = \sqrt{\frac{1520}{7}}$$

Homework: Pg. 169: #3,4,5,6,10.

So, the Falcons have a mean height of 165 cm and a S.D. of approx. 14.74 cm.

$$\begin{aligned}\text{Panthers: } \bar{x} &= \frac{166+163+168+161+165+166+166}{7} \\ &= \frac{1155}{7} \\ &= 165\end{aligned}$$

$$\begin{aligned}s &= \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + (x_3 - \bar{x})^2 + (x_4 - \bar{x})^2 + (x_5 - \bar{x})^2 + (x_6 - \bar{x})^2 + (x_7 - \bar{x})^2}{7}} \\ &= \sqrt{\frac{(166-165)^2 + (163-165)^2 + (168-165)^2 + (161-165)^2 + (165-165)^2 + (166-165)^2 + (166-165)^2}{7}} \\ &= \sqrt{\frac{(1)^2 + (-2)^2 + (3)^2 + (-4)^2 + (0)^2 + (1)^2 + (1)^2}{7}} \\ &= \sqrt{\frac{1+4+9+16+0+1+1}{7}} \\ &= \sqrt{\frac{32}{7}}\end{aligned}$$

≈ 2.14
So, the Panthers have a mean height of 165 cm and a S.D. of approx. 2.14 cm.

\therefore Both teams have a mean height of 165 cm but different standard deviations. The difference in S.D. means that some Falcon players are taller and some are shorter than Panther players.