1. a)

$$P(onedie\ shows\ a\ four) = \left(\frac{10}{36}\right) = \frac{5}{18}$$

b)

$$P(both\ \ show\ four) = \left(\frac{1}{36}\right)$$

c)

$$P(total\ is\ 6) = \frac{5}{36}$$

2. a)

$$P(the\ card\ is\ an\ ace) = \frac{\binom{4}{1}}{\binom{52}{1}} = \frac{1}{13}$$

b)

$$P(the\ card\ drawn\ is\ a\ balck\ ace) = \left(\frac{26}{52}\right)\left(\frac{4}{52}\right) = \frac{1}{26}$$

c)

$$P(the\ card\ is\ an\ ace\ or\ a\ king) = \frac{\binom{4}{1} + \binom{4}{1}}{\binom{52}{1}} = \frac{4+4}{52} = \frac{8}{52} = \frac{2}{13}$$

3. a)

$$P(the\ committee\ contains\ Adam\ and\ Steve) = \frac{\binom{23}{2} + \binom{23}{1} \times \binom{30}{1} + \binom{30}{2}}{\binom{55}{4}} = \frac{253 + 690 + 435}{341055} = \frac{1378}{341055} = \frac{2}{495}$$

b)

$$P(the\ committee\ consists\ of\ 3\ men\ and\ one\ woman) = \frac{\binom{25}{3}\binom{30}{1}}{\binom{55}{4}} = 20.25\%$$

Or

$$P(the\ committee\ consists\ of\ 3\ men\ and\ one\ woman) = \frac{4!}{3!\,1!}\left(\frac{25}{55}\right)\left(\frac{24}{54}\right)\left(\frac{23}{53}\right)\left(\frac{30}{52}\right) = 20.25\%$$

c)

$$P(\textit{the committee consists of at least one woman}) = 1 - P(\textit{the committee consists of } 4 \textit{ men})$$
$$= 1 - \left(\frac{25}{55}\right)\left(\frac{24}{54}\right)\left(\frac{23}{53}\right)\left(\frac{22}{52}\right)$$
$$\approx 0.9629$$
$$\approx 96.3\%$$

Or

$$P(\textit{the committee consists of at least one woman}) = 1 - \frac{\binom{25}{4}}{\binom{55}{4}}$$
$$\approx 0.9629$$
$$\approx 96.3\%$$

4. a) $P(\textit{he will get a matched pair}) = \left(\frac{10}{19}\right)\left(\frac{9}{18}\right) + \left(\frac{6}{19}\right)\left(\frac{5}{18}\right) + \left(\frac{3}{19}\right)\left(\frac{2}{18}\right)$

$$= \frac{5}{19} + \frac{5}{3 \times 19} + \frac{1}{3 \times 19}$$
$$= \frac{5}{19} + \frac{6}{3 \times 19}$$
$$= \frac{5}{19} + \frac{2}{19}$$
$$= \frac{7}{19}$$

Or

$$P(\textit{He will get a matched pair}) = \frac{\binom{10}{2} + \binom{6}{2} + \binom{3}{2}}{\binom{19}{2}}$$
$$= \frac{63}{19 \times 9}$$
$$= \frac{7}{19}$$

b) 100%

5. a)

$$P(\textit{all three are red}) = \frac{\binom{7}{3}}{\binom{11}{3}}$$
$$= \frac{35}{165}$$
$$= \frac{7}{33}$$

b)

$$P(\textit{all three are black}) = \frac{\binom{4}{3}}{\binom{11}{3}}$$
$$= \frac{4}{165}$$

c)

$$P(two\ are\ red\ and\ one\ is\ black) = \frac{\binom{7}{2}\binom{4}{1}}{\binom{11}{3}} = \frac{28}{55}$$

OR

$$P(two\ are\ red\ and\ one\ is\ black) = \frac{3!}{2!\,1!}\left(\frac{7}{11}\right)\left(\frac{6}{10}\right)\left(\frac{4}{9}\right) = \frac{28}{55}$$

6. Let A be the event that both cards drawn are not aces. Then,

$$P(A) = \left(\frac{48}{52}\right)\left(\frac{47}{51}\right) = \frac{564}{663}$$

And

$$P(A') = 1 - P(A) = 1 - \frac{564}{663} = \frac{99}{663}$$

$$Odds\ in\ Facour\ of\ A = \frac{P(A)}{P(A')} = \frac{564}{663} \times \frac{663}{99} = \frac{188}{33}$$

So, the odds in favour of not drawing an ace are 188 to 33.

7. Let A be the event that both balls drawn are red. Then,

$$Odds\ in\ Facour\ of\ A = \frac{P(A)}{P(A')} = \frac{\left(\frac{8}{13}\right)\left(\frac{7}{13}\right)}{1 - \left(\frac{8}{13}\right)\left(\frac{7}{13}\right)} = \frac{\frac{14}{39}}{1 - \frac{14}{39}} = \frac{14}{39} \times \frac{39}{25} = \frac{14}{25}$$

Therefore, the odds that both bolls are red are 14 to 25.

8. a) There are $6! = 720$ different arrangements of the letters of streET.
   b) There are $\frac{6!}{2!2!} = 180$ different arrangements of the letters of street.
   c) There are $\frac{7!}{3!2!} = 420$ different arrangements of the letters of *aaabbcd*.

9. a) T O T A WA
   There are $4! = 24$ different arrangements of OAWA.
   There are 2 T's, so there are $2! = 2$ arrangements.
   Therefore, 2x24=48 arrangements start with a T and the second letter is not a T.
   b) Treat TA as one group/section, TA OTWA .
   There are $\frac{5!}{2!} = 60$ arrangements that have TA side by side.

10. a) There are 10 letters in the word BLACKSMITH, so there are $10 \times 9 \times 8 \times 7 \times 6 = 30240$ possible arrangements.

Or

$$P(10,5) = \frac{10!}{5!} = 30240$$

   b) Since there are two vowels, A and I, then there are $2! = 2$ different possible arrangements.
   Since the word starts with a vowel, A or I, treat AI as one section, and treat C as one section, and treat B as one section. In that case, we are left with 7 letters. But, since we are making 5-letter words and since 3 letters (either A or I and C and B) are already in the 5-letter word, then we have to choose 2 letters from the 7 letters left. So there are 42 different possible ways of choosing the 2 letters.

$$P(7,2) = \frac{7!}{5!} = 42$$

   In total, there are 2x42=84 5-letter words in which B must be last, C in the middle and a vowel first.